

SPONTANEOUS SYMMETRY BREAKING IN COUPLED RING RESONATORS WITH LINEAR GAIN AND NONLINEAR LOSS

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Abstract: We present the study of the dynamics of a two-ring waveguide structure with space-dependent coupling, constant linear gain and nonlinear absorption. This system can be implemented in various physical situations such as optical waveguides, atomic Bose-Einstein condensates, polarization condensates, etc. It is described by two coupled nonlinear Schrödinger equations. For numerical simulations we take local Gaussian coupling (single-Gaussian and double-Gaussian). We find that, depending on the values of involved parameters, we can obtain several interesting nonlinear phenomena, which include spontaneous symmetry breaking, modulational instability leading to generation of stable circular flows with various vorticities, stable inhomogeneous states with interesting structure of currents flowing between rings, as well as dynamical regimes having signatures of chaotic behavior. In this paper, we only focused on consider phenomenon of spontaneous symmetry breaking in the case of space dependent coupling. The results show that in the case of a coupling between the two rings is a function of single-Gaussian symmetry breaking only between rings. In contrast, in the case of a coupling between them as a double-Gaussian function, the symmetry breaking occurs only in each ring, breaking the symmetry of the space dependent coupling.

1. Introduction

Spontaneous symmetry breaking is an important concept in many areas of physics. A fundamentally simple symmetry breaking mechanism in electrodynamics occurs between counter-propagating electromagnetic waves in ring resonators, mediated by the Kerr nonlinearity. In the nonlinear media, the symmetry breaking phenomenon has been studied in many different models. The spontaneous symmetry breaking of soliton and phase transitions trapped in a double-channel potential [1]. Recently, scientists have focused on studying for double-channel, the symmetry breaking not only between two channels but also in each channel [2]. In the ring resonators, the earliest paper studied discontinuous behavior in the onset of spontaneous symmetry breaking, indicating divergent sensitivity to small external perturbations [3].

Coupled microrings are a natural laboratory studying different phenomena in both optics and Bose-Einstein condensates (BECs). In optics, they are used for nonreciprocal devices [4], switches [5], loss control of lasing [6] and ring lasers [7]. In the case of atomic Bose-Einstein condensates the ring-shaped geometry allows to obtain persisting superfluid currents and consider their interaction with various types of the defects. It is

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reason why dynamics of atomic BECs loaded in toroidal traps have been intensively explored experimentally [8] and studied theoretically both in the full three-dimensional toroidal geometry [9] and within the framework of the reduced quasi-one-dimensional Gross-Pitaevskii equation (GPE) with periodic boundary conditions [10]. Coupled non-Hermitian microcavities are also used for the study of chiral modes in exciton-polariton condensates [11] as well as for modeling coupled circular traps for BEC, where gain corresponds to adding atoms while nonlinear losses occur due to inelastic two-body interactions.

Additionally, for the cyclic geometry, many applications of the system appear due to different physical properties, for example the (cubic) nonlinearity. All considered problems are based on the same mathematical model. In optical systems, the Kerr nonlinearity is as a result of the fact that the refractive index of the medium depends on the intensity of the light, and in the mean-field theory of condensates, it appears due to two-body interactions.

In this paper we consider a model of two coupled ring waveguides with constant linear gain and nonlinear absorption with space-dependent coupling. The coupling between two waveguides is single-Gaussian or double-Gaussian. As it has been emphasized above, this system can be implemented in various physical situations such as optical waveguides, atomic Bose-Einstein condensates, polarized condensates, etc. It is described by two coupled nonlinear Schrödinger equations. It has been found that depending on the values of involved parameters, we can obtain several interesting nonlinear phenomena, which include spontaneous symmetry breaking. We concentrated on studying symmetry breaking of states between two waveguides and in each waveguide.

2. The Model

In the present study, we consider a model described by two coupled nonlinear Schrödinger equations with gain and nonlinear loss (depending on the applications, they also can be termed Gross-Pitaevskii or Ginzburg-Landau equations), which is written down in scaled dimensionless units as following:

$$\begin{cases} i\partial_t\psi_1 = -\partial_x^2\psi_1 + i\gamma\psi_1 + (1 - i\Gamma)|\psi_1|^2\psi_1 + J(x)\psi_2 \\ i\partial_t\psi_2 = -\partial_x^2\psi_2 + i\gamma\psi_2 + (1 - i\Gamma)|\psi_2|^2\psi_2 + J(x)\psi_1. \end{cases} \quad (1)$$

Obviously, ψ_1 and ψ_2 are the fields in the first and second waveguides, γ is the linear gain and Γ is the nonlinear loss. Both are considered as constants along the waveguides, and $J(x)$ is the position depending on coupling.

The first application of model (1) can be found in a reference [12], where the discussed rings coupled homogeneously, i.e., where it was assumed that $J(x)$ is constant. The model with local single-Gaussian coupling modulation has been considered [13]. In current work we continue study originated in publication [13] and we also introduce new model in which the $J(x)$ is local double-Gaussian coupling modulation.

In numerical calculations we assume, without loss of generality, that $x \in [-\pi, \pi]$. This implies periodic boundary conditions for both channels: $\psi_i(x, t) = \psi_i(x + 2\pi, t)$, and the coupling function $J(x)$ is concentrated in a certain region of the rings. In

particular, for numerical simulations, we shall consider local Gaussian coupling in the following form in two cases:

$$J(x) = \frac{J_0}{\sqrt{\pi}a} \exp\left(-\frac{x^2}{a^2}\right), \quad (2)$$

$$J(x) = \frac{J_0}{\sqrt{\pi}a} \left\{ \exp\left(-\frac{\left(x-\frac{\pi}{2}\right)^2}{a^2}\right) + \exp\left(-\frac{\left(x+\frac{\pi}{2}\right)^2}{a^2}\right) \right\}, \quad (3)$$

where a is the width of the coupling, while J_0 characterizes the coupling strength. Our results are not sensitive to the particular shape of the wavefunction, as we have checked using super Gaussian functions raised to high power n .

For all applications mentioned in the Introduction, the meaning of the variable x is an angle defining a point on the circumference. The functions $\psi_{1,2}$ are envelopes of the field distributions (see, e.g., [14] for optical resonators and the total fields [15] for BECs applications). Most of the results found in the present study are numerical. For the uncoupled case ($J_0=0$), one can find stable background solutions in the form:

$$\psi_{1,2}(t) = \sqrt{\frac{\gamma}{\Gamma}} e^{-\frac{\gamma}{\Gamma}t}. \quad (4)$$

These solutions are implemented in both channels and then are used as initial state in our numerical investigations.

When the rings become coupled, due to the interplay between gain and nonlinear absorption, they lead to modulation instability. In the case of constant coupling in [12], two distinct classes of solutions have been found analytically: symmetric, characterized by $\psi_1 = \psi_2$, and anti-symmetric with $\psi_1 = -\psi_2$. The anti-symmetric solutions are always stable, whereas symmetric ones are usually unstable. Therefore, we decided to perform numerical studies using the symmetric state as the initial condition.

We found various final states obtained after long time evolution: different types of the solutions included, stationary anti-symmetric, symmetric and asymmetric solutions and stationary time dependent states. In particular, when coupling is spatially dependent and relatively narrow (small in comparison with the ring length), the results can be stable, stationary states (including those with broken symmetry), or time dependent limit cycles states.

The initial state with small perturbation imposed is in the form:

$$\psi_{1,2}(x, t = 0) = \sqrt{\frac{\gamma}{\Gamma}} (1 + \beta \sin(kx)), \quad (5)$$

where the perturbation β was typically of the order of 10^{-2} . In our simulations, we took the value of the loss $\Gamma=1$, the coupling strength $J_0=1.5$, the width of Gaussian coupling is narrow $a = 0.01$ in two cases (single-Gaussian coupling and double-Gaussian coupling) and change the linear gain coefficient γ . We noticed that the results do not depend on particular values of the amplitude of the perturbation β or perturbation wavenumber k . All simulations were performed using the so called ‘‘Pseudospectral method’’ and ‘‘Split-Step-Fourier method’’ [16].

3. Results and discussion

3.1. Stationary solution

We considered the cases when we fix the coupling strength $J_0 = 1.5$, the nonlinear loss $\Gamma=1$, the width of Gaussian coupling $a = 0.01$ (namely narrow coupling) and change the gain parameter γ . Note that we always start from the perturbed symmetric state given in equation (5). For the model with the single-Gaussian coupling, the dark soliton state appeared when the $\gamma < 0.70$ and when the $\gamma \gtrsim 1.05$, we obtained anti-symmetric stationary solutions with one-peak. It was interesting that we obtained oscillation asymmetric states when the linear gain was in the range $0.7 \lesssim \gamma < 1.05$ (we will study in more detail in section 2 and section 3). For the model with the double-Gaussian coupling we also obtained the same results. When the $\gamma < 0.38$, symmetric stationary solution with one-peak is found and asymmetric stationary solution with two-peak obtained with $\gamma \gtrsim 0.72$. When $0.38 \lesssim \gamma < 0.72$, we also obtained oscillation asymmetric states.

In addition, we also considered cases of two above models when the width of Gaussian coupling is broad (which will briefly called broad coupling, here we choose the width of Gaussian function $a = 1$), specifically, for both the two models with the single-Gaussian coupling and double-Gaussian coupling, the parameters areas of the linear gain γ in which leads to the oscillation asymmetric states is smaller in comparison with narrow coupling case. The other regions of the linear gain γ gave us stationary states.

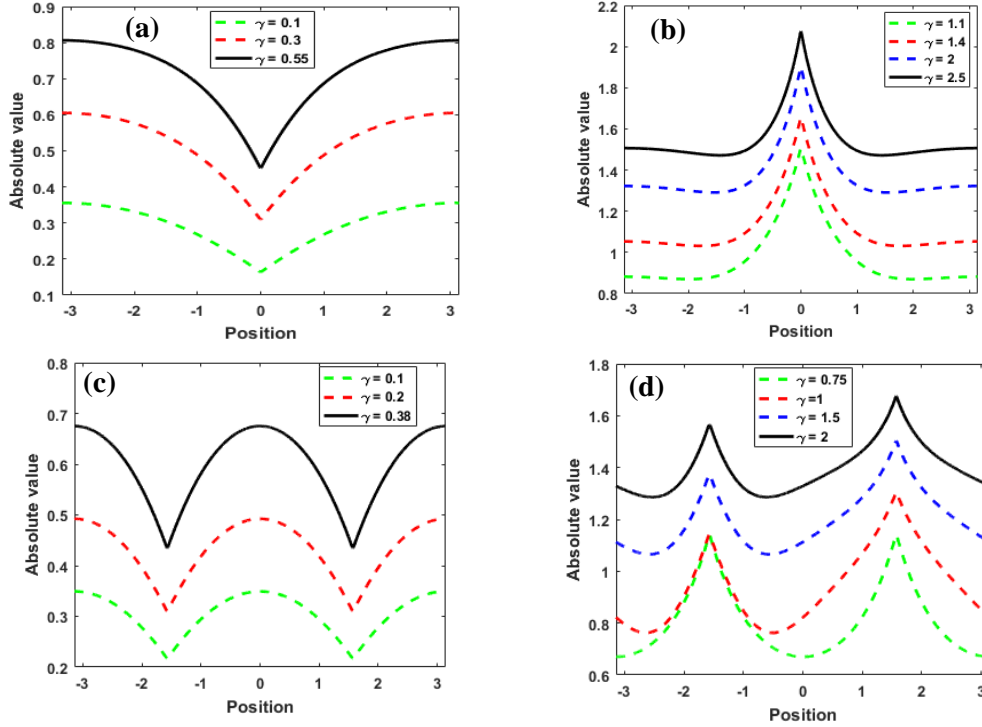


Fig. 1: Absolute values of stationary states after propagation long time in the coupled double-ring system (1) obtained for the initial conditions (5) with $\Gamma=1$, $J_0 = 1.5$, $a = 0.01$.

Fig. (a): Symmetric dark solitons calculated for different small linear gain for the single-Gaussian coupling. Fig. (b): Antisymmetric states calculated for different small linear gain for the single-Gaussian coupling. Fig. (c): Symmetric solutions calculated for different small linear gain for the double-Gaussian coupling. Fig (d): Asymmetric solutions calculated for different linear gain for the double-Gaussian coupling.

3.2. Spontaneous symmetry breaking in model with single-Gaussian coupling

3.2.1. Narrow coupling case

In this section, we considered symmetry breaking in single-Gaussian model and fix the parameters as: nonlinear loss $\Gamma = 1$, the coupling strength $J_0 = 1.5$, the width of Gaussian coupling $a = 0.01$, whereas we change linear gain γ . We used Pseudospectral Method to simulate propagation of wave function for different values of the linear gain γ with initial symmetry state given in Eq. (5). As it has been mentioned in section 1, when $\gamma \approx 0.55$ we obtained symmetry stationary solution and see that spontaneous symmetry breaking did not appear, clear that in Fig. 2(a₁) present absolute values of other states as a function of time, Fig. 2(b₁) show the curve of norm N_1 practically coincides with the curve of the norm N_2 corresponding to the linear gain $\gamma = 0.55$. When the linear gain is in the domain $0.55 < \gamma < 1.05$, we see that spontaneous symmetry breaking between two rings occurred. The symmetry breaking was illustrated by difference between norm N_1 and N_2 (with $N_i = \int_{-\infty}^{+\infty} |\psi_i|^2 dx$). This difference can be seen clearly in Fig. 2(b₂) and 2(b₃). As we has been mentioned previously, when $0.7 \lesssim \gamma < 1.05$ the propagation of wavefunction oscillated with different frequencies that is, there is symmetry breaking phenomenon in that range. We also found that the spontaneous symmetry breaking did not occur when the gain increased $\gamma \gtrsim 1.05$.

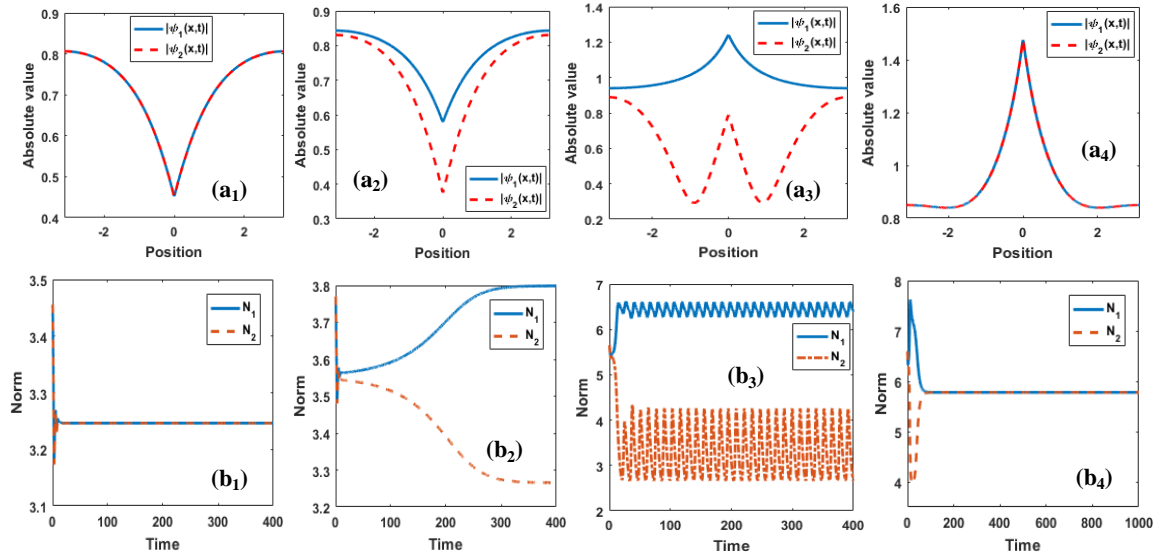


Fig. 2: Top row: Absolute values of other states, Fig. (a₁) and (a₄) are stationary states with the $\gamma = 0.55, 1.05$, respectively, do not symmetry breaking; Fig. (a₂) and (a₃) are asymmetric states with the $\gamma = 0.6, 0.9$, respectively. Bottom row: Norm values of wavefunctions, Fig. (b₁) and (b₄) have $N_1 = N_2$ that meaning do not occur symmetry breaking between two rings; Fig. (b₂) and (b₃) have $N_1 \neq N_2$ that meaning occur

symmetry breaking between two rings. The under figures are corresponding to the up figures about parameters. All above case use parameters: $\Gamma = 1, J_0 = 1.5, a = 0.01$.

3.2.2. Broad coupling case

In the opposite limit, when the range of the coupling is comparable to the length of the ring (but not uniform yet), we also observe the spontaneous symmetry breaking, and we classify them according to the (increasing) value of linear gain. We present results for $a=1$, performed simulation almost through all the range of γ and obtained the results as below. When the linear gain $\gamma \lesssim 0.35$, dynamics leads directly to the symmetric stationary states. The spontaneous symmetry breaking occurred with $0.35 < \gamma \lesssim 0.51$. When the $\gamma \gtrsim 0.51$, dynamics leads directly to the anti-symmetric stationary states. The oscillation asymmetric states occurred when the linear gain belongs to the domain $0.42 \lesssim \gamma \lesssim 0.50$.

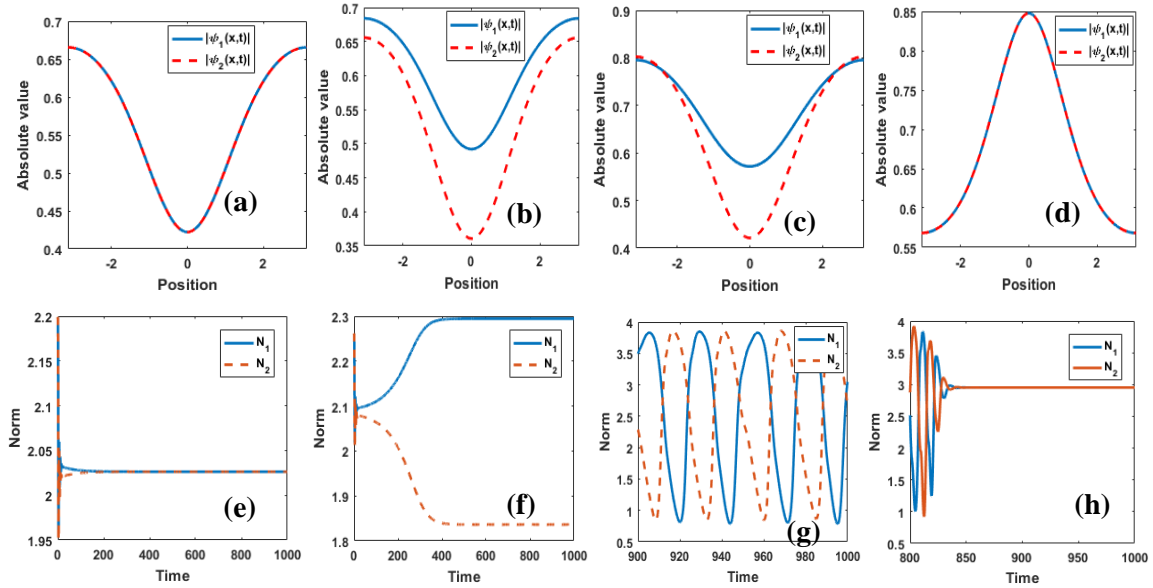


Fig. 3: *Top row: Absolute values of other states, Fig. (a) and (d) are stationary states with the $\gamma = 0.35, 0.51$, respectively; Fig. (b) and (c) are asymmetric states with the $\gamma = 0.36, 0.50$, respectively. Bottom row: Norm values of wavefunctions, Fig. (e) and (h) have $N_1 = N_2$ that meaning do not occur symmetry breaking between two rings; Fig. (f) and (g) have $N_1 \neq N_2$ that meaning occur symmetry breaking between two rings. The under figures are corresponding to the up figures about parameters. All above case use parameters $\Gamma = 1, J_0 = 1.5, a = 0.01$.*

In summary, in this section we have examined the symmetry breaking between two rings for the model with single-Gaussian coupling with both two cases: narrow coupling and broad coupling. The symmetry breaking between two rings occurred in two cases. Each case had different parameter regions of linear gain. The parameter value regions for γ in broad coupling case are smaller than for narrow coupling case.

3.3. Spontaneous symmetry breaking in model with double-Gaussian coupling

3.3.1. Narrow coupling case

We next considered the model of double-Gaussian coupling. As in the model of single - Gaussian coupling we also fix the nonlinear loss $\Gamma = 1$, coupling strength $J_0 = 1.5$, width of Gaussian coupling $a = 0.01$ and change linear gain γ . We obtained the results that the spontaneous symmetry breaking between two rings did not occurred but in each ring spontaneous symmetry breaking occurred. The symmetry breaking in this case was featured by asymmetric ratio:

$$\Theta_i = \frac{\int_0^{+\infty} |\psi_i|^2 dx - \int_{-\infty}^0 |\psi_i|^2 dx}{\int_{-\infty}^{+\infty} |\psi_i|^2 dx}. \tag{6}$$

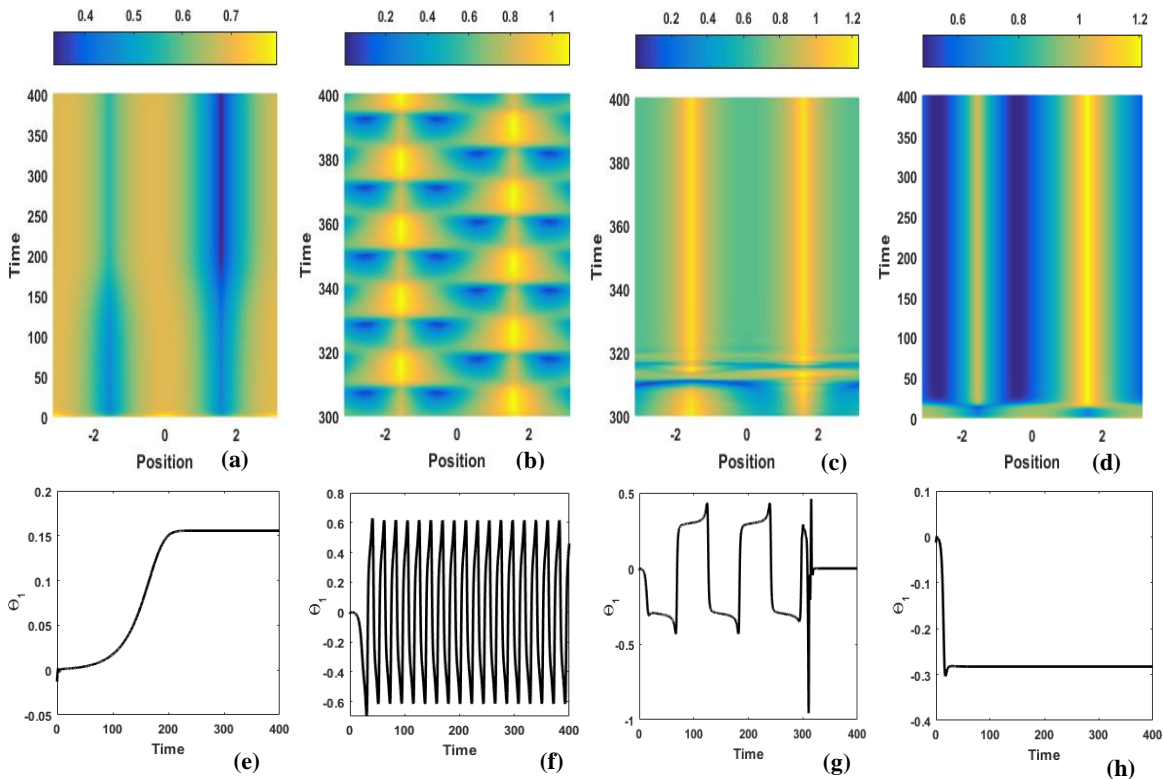


Fig.4: Top row: Contour plots of absolute values of the propagated wavefunction ψ_1 in the stationary regime for four different linear gain, from left to right corresponding to $\gamma = 0.4, 0.6, 0.77, 0.78$ and the fixed width $a = 0.01$. Bottom row: The asymmetric ratio in first wavefunction, defined in equation (6). The under figures are corresponding to the up figures about parameters. All above case used parameters $\Gamma = 1, J_0 = 1.5, a = 0.01$.

In narrow coupling case, when $\gamma \lesssim 0.38$ or $0.72 < \gamma < 0.77$ we obtained the stationary states and symmetry breaking which are not occurred. The oscillation asymmetric states appeared in range of the linear gain $0.38 < \gamma < 0.72$, the asymmetric states of course there is symmetry breaking. Final domain is $\gamma > 0.77$, the asymmetric states appeared in this range and have symmetry breaking. The Fig. 4(e) and 4(h) show the Θ_1 of wavefunction ψ_1 at $\gamma = 0.4$. We see that it is a constant different from zero

which proves that here is stationary state and have symmetry breaking. The Fig. 4(b) and 4(f) show oscillation asymmetric states. In this case, we see that the asymmetric ratio oscillation varies around the zero and circulates with respect to time, implying that the symmetry breaking is periodicity.

3.3.2. Broad coupling case

Now we considered the broad coupling case (here we choose the width coupling $a=1$) for the double-Gaussian coupling. For broad coupling we obtained the parameter regions $\gamma \lesssim 0.07$ and $\gamma \gtrsim 1.1$ in which the stationary states obtained, whereas when $0.07 < \gamma < 0.19$ the oscillation asymmetric states obtained, Fig. 5b, 5c present contour plots of absolute values of the propagated wave function ψ_1 at $\gamma = 0.08$ and $\gamma = 0.18$, respectively. The asymmetric states also found in range of linear gain $0.19 < \gamma < 1.1$, have symmetry breaking phenomenon in each ring. The figures 5 (a), (b), (c), (e), (f), (g), (h) are the cases of threshold points which have shifted from given state to other state. Specifically, when $\gamma = 0.07$ (look at Fig. 5 (a), (e)), we found the asymmetric stationary states while the oscillation asymmetric states are obtained with $\gamma = 0.08$. The results were the same that the oscillation asymmetric states obtained with $\gamma = 0.18$ and asymmetric stationary states with $\gamma = 0.19$.

Thus in model with the double-Gaussian coupling we only found the symmetry breaking in each ring, whereas we did not find the symmetry breaking between two rings.

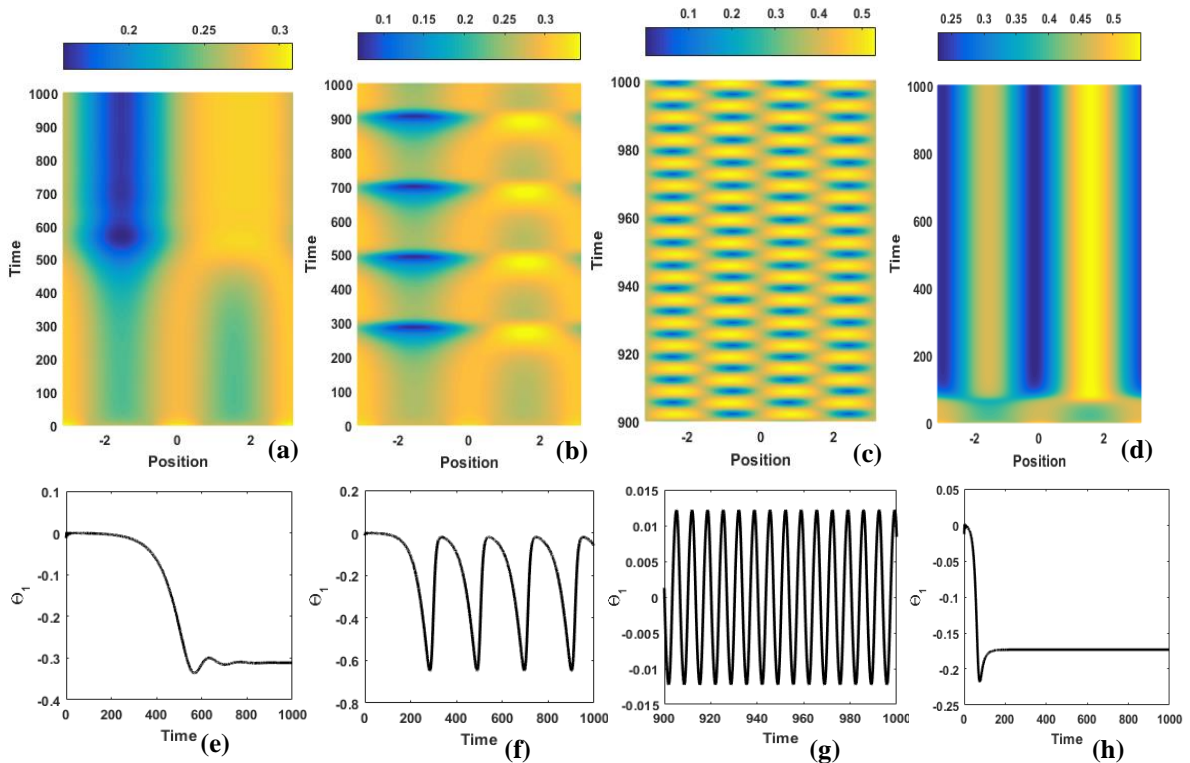


Fig. 5: Top row: Contour plots of absolute values of the propagated wavefunction ψ_1 in the stationary regime for four different linear gain, from left to right corresponding to $\gamma = 0.07, 0.08, 0.18, 0.19$ and the fixed width $a = 1$. Bottom row: The asymmetric ratio between of two wavefunctions, defined in equation (6). The under figures are corresponding to the up figures about parameters. All above case use parameters $\Gamma = 1, J_0 = 1.5, a = 1$.

4. Conclusion

In this paper, we have studied the symmetry breaking both for the single-Gaussian and double-Gaussian model for nonlinear loss $\Gamma = 1$ and coupling strength $J_0 = 1.5$, with changing linear gain γ , in two cases: narrow coupling and broad coupling. We found the results that for same linear gain parameter regions different kind of symmetry breaking exists. For the model of single-Gaussian coupling the symmetry breaking occurred between two rings while these phenomena occurred in each ring for the model of double-Gaussian coupling. In addition, we found parameter areas where the oscillation asymmetric states, symmetric stationary states, anti-symmetric stationary solutions appeared. Specially, the dark soliton state appeared in the model of single-Gaussian coupling. Further studies of this system are planned and they may bring some new and exciting results.

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TÓM TẮT

SỰ PHÁ VỠ ĐỐI XỨNG TỰ PHÁT TRONG BỘ CỘNG HƯỞNG VÒNG LIÊN KẾT VỚI KHUẾCH ĐẠI TUYẾN TÍNH VÀ MẮT MẮT PHI TUYẾN

Chúng tôi nghiên cứu mô hình của hai vòng ống dẫn sóng với khuếch đại tuyến tính, hấp thụ phi tuyến không đối và liên kết phụ thuộc không gian. Hệ này có thể thực hiện trong các lĩnh vực vật lý khác nhau như trong ống dẫn sóng quang học, nguyên tử ngưng tụ Bose-Einstein, sự ngưng tụ phân cực, v.v... Hệ được miêu tả bởi hệ phương trình Schrödinger. Đối với kết quả mô phỏng số, chúng tôi sử dụng liên kết dạng hàm Gauss cục bộ (dạng đơn Gauss và hai Gauss). Chúng tôi tìm thấy rằng tùy thuộc vào các giá trị tham số liên quan, thu được một số hiện tượng thú vị bao gồm sự phá vỡ đối xứng tự phát, sự bất ổn định dẫn tới các dòng tuần hoàn với các xoáy tùy ý, trạng thái không đồng nhất với cấu trúc thú vị của các dòng giữa các vòng, cũng như chế độ động học có dấu hiệu của trạng thái hỗn loạn. Trong bài báo này, chúng tôi chỉ tập trung chủ yếu vào hiện tượng phá vỡ đối xứng tự phát. Kết quả cho thấy rằng trong trường hợp liên kết giữa hai vòng là hàm đơn Gauss sự phá vỡ đối xứng chỉ xảy ra giữa các vòng với nhau. Ngược lại trong trường hợp liên kết giữa chúng là hàm hai Gauss thì sự phá vỡ đối xứng lại chỉ xảy ra trong mỗi vòng, phá vỡ tính đối xứng của liên kết không gian.